Broadcasting on a highway - ad hoc warning systems

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1 Preface

A reliable information system on traffic conditions and presence of exceptional situations on the road would enhance the overall speed of travelling and positively affect security. This is especially true when considering increasing crowdedness of highways in European countries and high car speeds.

Our goal is to introduce a system based on ad-hoc communication devices located on cars. The network has to provide drivers with information about situations ahead of them – so that they can take immediate action when a traffic jam is suddenly appearing several hundreds of meters in front or take a detour if there is a great jam several kilometers ahead.

The system must operate in conditions typical for a highway environment. These are:

- cars can move with high speeds, the car speed may vary, and it is not trivial to predict it,
- the terrain form and other cars can have substantial influence on the quality of communication links,
- energy cost, usually very significant in ad-hoc environments, can be ignored.

The paper is organized as follows. In the introduction we discuss the model within which our system works. This covers basically the assumptions about the underlying communication hardware and its properties. In the second subsection we describe previous work, which deals with leader election, size approximation and broadcasting within ad-hoc networks – these procedures play the fundamental role in our system. Later we present a general description of our system, decomposing it into several algorithms.

In the next chapters we focus on the particular algorithms used by our system: size approximation, leader election and broadcasting.

After presenting a basic system suitable for very cheap hardware, we extend our model slightly and present a modification which is suitable for environments where traffic is light. This modified system requires more effort on hardware side for proper functioning.

2 Introduction

2.1 Model

We assume that a radio communication channel is available for communication in our system. In order to get robust algorithms that are resistant to
several technical problems we assume that collision detection is not available.

The stations are synchronized so that we may assume that they have a common clock. Time is divided in atomic slots (a-slots), and each station can transfer a one-bit message in a a-slot. Thus, depending on whether only one or more stations transmitted in this slot, the output of one slot can be ZERO, ONE or NOISE. Stations transmitting can listen to the channel while transmitting, thus checking whether their transmission has been successful. Note that we do not adhere to a situation for ad hoc networks described in 802.11 IEEE standard. The reason is that on a car we can in fact deploy not one but many communication devices.

Each transmission has a limited range so that only stations within this distance can clearly receive the message. The range is adjustable by each station for each transmission, limited only by the maximum range $r_{\text{max}}$ of the radio unit.

With each transmission range $r$ we associate an interference range of $\alpha \cdot r$, where $\alpha$ is a constant. Each station within distance $r < d \leq \alpha \cdot r$ from a transmitting station receives only NOISE. This assumption models the physical property that radio links interfere with other links on longer distances than they in fact work on.

The number of transmission slots available per second is limited. In order to model communication link reliability we assume that with a certain probability, a station may fail to receive the message, i.e. instead of a proper one-bit message, the station receives NOISE though the transmission would be normally successful. We denote the upper bound for this probability as $p_r$.

We assume that the devices used to construct a network are equipped with (pseudo)random bit generators with stochastic properties sufficiently close to random unbiased sources.

2.2 Relation to previous work

Current research on ad-hoc networks focuses on high-level algorithm design. Most of the created algorithms have very good asymptotical running times. Significant achievements have been obtained with regard to the quality of solutions in terms of diverse complexity measures. However, the complexities are often studied in terms of asymptotic behaviour.

An evaluation of the available algorithms for low level ad-hoc communication organization shows that they are not well-suited for systems with low communication bandwidth possibilities. A good overview over broadcasting algorithms can be found in [4], though the algorithms presented there rely on an underlying MAC structure and are not aware of node mobility.
Similar broadcasting algorithms are considered in [7] but they lack the ability to support station mobility in the model.

Approximation size algorithms presented in previous works has good theoretical results. Their running times are poly-logarithmic [10], they are energy efficient [3], but due to large constants they are not applicable in our system.

Our novel approach is to consider very low communication bandwidth, very high node mobility and realistic physical assumptions about hardware possibilities. Moreover, we present a complete system and do not focus only on one part of it.

2.3 System description

2.3.1 Sector organization

We divide a highway into sectors, uniquely numbered with increasing integers, so that each station can compute its distance to a given sector and determine whether it lies in the driving direction of the station. The sectors are aligned on one line, so each sector has two neighboring sectors, one in the driving direction and one in the opposite. We make a simplification by assuming the width of a sector to be zero. The length of each sector is $r_{\text{max}}/2$, so that the maximum distance between two stations in neighboring sectors is $r_{\text{max}}$, and the maximum distance between two stations in one sector is $r_{\text{max}}/2$. The upper limit for the number of stations $N$ in one sector is known (for the obvious reason that the number of cars on a certain part of the road is physically limited by car sizes).

Our algorithms use transmission ranges of either $r_{\text{max}}$ for communication between sectors or $r_{\text{max}}/2$ for communication within one sector. All stations operate using the same communication channel so we obtain an interference area equal to $\alpha r$ with $r$ being the transmission range.

2.3.2 Algorithms

The system is composed of 3 different algorithms. The first two: size approximation (or SA for short) and leader election (LEA for short) build a framework for the broadcasting algorithm (BCAST) to work properly.

The output of the size approximation algorithm is an approximation of the number of nodes (cars) within each sector. This value is then used by the leader election algorithm to elect a leader for each sector for some short time. Because of high node mobility and communication channel reliability, the leaders within a network are changed very often and the number of nodes is recomputed often, too. This poses high requirements on the time-efficiency
of both algorithms.

The broadcasting algorithm plays the key role in the system. It is responsible for passing a message about traffic jam position from its source to other sectors. Within the broadcast algorithm, only the current leader in each sector plays an active role – all other stations are passive (where active means that it is allowed to send messages). This implies that the leader election algorithm must elect a leader successfully for the broadcasting to work properly.

Whenever a station receives a message (via the broadcasting algorithm), it decodes it and can notify the driver or the on-board navigation system about the event detected on the road. The message should be long enough to accommodate information about event location, so that the navigation system can decide whether to take a detour and the driver can be notified about the distance left to a traffic jam.

2.3.3 Time organization

Available communication time is statically divided into groups of slots, the so called g-slots. In each g-slot, composed of many a-slots, only one type of algorithm is allowed to transmit. Figure 1 presents the division of one second into distinct g-slots. The different g-slots are labeled SA, LEA and BCAST on the figure, denoting the algorithm which is executed within them.

![Figure 1: Division of available time in g-slots](image)

Each second is divided into g-slots in the same way. As shown on fig. 1, each g-slot for BCAST is preceded by a g-slot for LEA. This means that for each g-slot, the leader election outputs a recent leader. Within one g-slot for BCAST, each sector can transmit exactly one message. Within one g-slot for LEA each sector can elect a leader and within one g-slot for SA, the size approximation algorithm can approximate the number of stations within a sector. This implies that g-slots have different lengths (they consist of different numbers of a-slots).

Note that algorithms running in neighboured sectors at the same time could interfere with each other. To solve this problem, each g-slot is divided
into gs-slots. Only sectors which are far enough in order not to interfere with each other can transmit within the same gs-slot. As short-range communication interferes with less sectors than long-range communication, the division can be differentiated, depending on the algorithm and its transmission range.

Figure 2: Transmission and interference for \( r_{\text{max}}/2 \)-strong communication, where \( r_{\text{max}}/2 \) is also the length of a sector

Figure 2 shows the transmission and interference ranges of a node, located in sector \( x \). The node is transmitting with range \( r_{\text{max}}/2 \) and is located at the upper limit of sector \( x \). Within this illustration, the interference ratio has been chosen to be \( \alpha = 2 \), so that with each transmission at most 2 sectors in one direction are jammed.

We number gs-slot within one g-slot with increasing numbers, starting from 0. A sector is allowed to transmit in a gs-slot if and only if the sector’s ID is congruent to the gs-slots number mod 3. Figure 3 shows the division of g-slots in gs-slots.

The effect is that two sectors which could potentially interfere with each other because they lie near to each other, are not allowed to transmit within one a-slot. The considered transmission range is \( r_{\text{max}}/2 \) and this transmission range is used by the size approximation and leader election algorithms. So this division can be applied only to the SA and LEA algorithms.

For broadcasting, the transmission range is \( r_{\text{max}} \) and so the interference region is double as big as for the other algorithms (see Figure 4). So, for broadcasting, each g-slot must consist of 6 different gs-slots and the sector’s ID must be congruent to the gs-slots number mod 6.
2.3.4 Message generation

We let the leader of each sector to check at the beginning of each second check whether a new message should be broadcasted at this moment. This knowledge can be obtained by sensors within the car. Then if there is a need for transmitting a message, this is performed at the first g-slot for BCAST in a second.

3 Network size approximation

3.1 Introduction

In the current section we present solutions of the problem of estimating the number of stations in a single-hop radio network. We assume also that upper bound on number of stations in a sector is known to each station. We do not demand serial numbers or IDs.

In contrast to other solutions we do not pay attention to energy cost of our algorithms. Our aim is to get algorithms which are fast and reliable. We are interested in achieving good results for small number of stations, without considering asymptotical complexity.

Various algorithms for size approximation are compared in [3], the best one achieves $O(\log^2 n)$ time complexity with constants $> 2$. For our scenario, when number of stations in the sector is limited by $N_{max} = 150 \approx 2^8$ it turns out that this algorithm requires $2 \cdot 8^2 \approx N_{max}$ time slots. Our algorithm is more off for small networks than the other algorithms. We show linear time algorithm with small constants, smaller than 1.

**Definition 1** Let $\frac{1}{c}N \leq n \leq cN$, for some real number $c$, where $N$ is exact number of stations and $n$ is approximated value. Then, $c$ is called an approximation ratio for the size approximation problem.
We do not consider size approximation as separate problem. It will be used as a subprotocol in the whole system, especially in leader election algorithm. The more time we allow approximation algorithm to run, the better approximation ratio we get. But reserving more time for size approximation gives us less time for leader election and broadcasting. But there is also another tradeoff. The better approximation we have, less time we need for leader election, and we increase probability that leader election algorithm succeeds.

3.2 General remarks

In following paragraphs we assume that a maximum number of stations in a sector is equal to $N_{max}$, number of stations $N$ and, as a result of the algorithm each station knows approximation of $N$, $n$. 
We consider no-collision detection (no-CD) model. But we assume that every station has receiver and transmitter. Thanks to that, every station can determine whether its transmission was successful or not. It is achieved as follows: when station sends a message, it also listens to the channel. If it received the same message it would known that transmission was correct.

No collision detection means that only stations which have been transmitting can tell if there was a collision (in a way described above). It means that a station which does not transmit cannot distinguish the cases that the none of stations is transmitting in in a slot (it hears noise) or more than one station is transmitting (it also hears noise).

In all our algorithms we extensively use the fact that transmitting stations know if there was a collision or not.

3.3 First approach - BSAA

Here, we introduce a basic algorithm which we modify in following paragraphs to achieve desired properties.

Let proper (or successful) transmission be a transmission which is not scrambled. We divide stations in two groups: active stations and inactive stations. Inactive stations are those which are allowed to listen only. Active stations listen, but they can also transmit some messages.

**Algorithm 1 Basic Size Approximation - (BSAA)**

Algorithm runs in $f$ rounds. A round $j$, consists of $M_j$ time slots. At the begin of the algorithm, all stations are active. All stations (both active and inactive) listen all the time.

Before a round $j$, each active station $i$ (station, which takes part in the $j$-th round = is active in the $j$-th round), chooses independently at random time slot $T_{i,j} \in 1, \ldots, M_j$.

Every (active) station sends single bit message in time slot $T_{i,j}$. When there are no collisions with other transmissions ($T_{i,j} \neq T_{k,j}$, $i \neq k$), then the message is received properly by every station.

Every station counts number of proper transmitions in each round.

A station which has succeed (its transmition was successful) becomes inactive, which means, that it does not actively participate in following rounds, it only listens. Active stations, which had unsuccessful transmissions in a round $j$, stay active in the round $j+1$.

After $f$ rounds, every station computes approximation of the number of stations as the number of successful transmissions from all rounds.

In Theorem 2 we prove expected approximation ratio of one round of our
algorithm. We show that the number of rounds affects approximation the ratio.

The only information which stations get during the algorithm, is the number of successful transmissions in each round. Let $Z_j$ denote the number of successful transmissions in the $j$-th round.

**Lemma 1** Let $N_j$ denote the number of active stations in the $j$-th round. Let $M_j$ be the number of time-slots of the $j$-th round. Then the expected number of new inactive stations after the $j$-th round is $N_j(1 - 1/M_j)^{N_j-1}$

**Proof:** Every station chooses at random one of time slots and sends single bit message in it. Let $T_{i,j}$ denotes number of time slot chosen by station $i$. It means that $P[T_{i,j} = k] = \frac{1}{M_j}$, for $k = 1, 2, ..., M_j$.

Let $Y_{i,j}$ denotes the number of stations which have chosen the $i$-th time slot. Then

$$P[Y_{i,j} = l] = \binom{N_j}{l} \left(\frac{1}{M_j}\right)^l \left(1 - \frac{1}{M_j}\right)^{N_j-l} \quad (1)$$

Let $Z_{i,j} = [Y_{i,j} = 1]$ that is $Z_{i,j} = 1$ if and only if there is one station which has chosen to transmit its message in the $i$-th time slot. $Z_{i,j}$ is equal to 0 when none or more than one station have chosen slot $i$.

$$P(Z_{i,j} = 1) = P(Y_{i,j} = 1) = \frac{N_j}{M_j} \left(1 - \frac{1}{M_j}\right)^{N_j-1} \quad (2)$$

Let $Z^j = \sum_{i=1}^{M_j} Z_{i,j}$. Then, by the linearity of the expected value, we get

$$E[Z^j] = E[\sum_{i=1}^{M_j} Z_{i,j}] = \sum_{i=1}^{M_j} \binom{N_j}{l} \left(\frac{1}{M_j}\right)^l \left(1 - \frac{1}{M_j}\right)^{N_j-l} = N_j \left(1 - \frac{1}{M_j}\right)^{N_j-1} \quad (3)$$

Expectation of $Z^j$ is equal to the number of stations which successfully transmitted their messages in the $j$-th round. And it is (by definition) equal to the number of stations, which become inactive.

□

Consider function $\mu(m, n) = n(1 - \frac{1}{m})^{n-1}$ equal to the expected number of successful transmissions (counted stations) in one round, when there are exactly $n$ active stations in the given round and there are $m$ time slots reserved for that round. It is easy to check that $\mu(m, n)$ is increasing function for fixed $n$ and variable $m$. The price we pay for higher number of success is longer running time. Let us calculate optimal running time (number of time slots) for given number of stations.
Let \( p(m, n) \) be the function

\[
p(m, n) = \frac{\mu(m, n)}{m} = \frac{n}{m} \left( 1 - \frac{1}{m} \right)^{n-1}
\]  

(4)

We are interested in finding the minimum of that function, for given \( n \). Let’s calculate the derivative of function \( p \):

\[
p'_m(m, n) = \frac{1}{m^2} \left( n - \frac{1}{m} \left( 1 - \frac{1}{m} \right)^{n-2} - \left( 1 - \frac{1}{m} \right)^{n-1} \right)
\]  

(5)

Solving equation \( p'_m(m, n) = 0 \) we get that \( m = n \). Thanks to that result, we know how to set the number of time slots per round to achieve the best time efficiency.

Let’s look at the expected value of \( Z^i \) as a function of \( N_i \) and \( M_i \).

\[
f(x) = \frac{\mu(x)}{M}
\]  

(6)

And

\[
f(x) = x \left( 1 - \frac{1}{M_i} \right)^{N_i-1} = x \left( 1 - \frac{x}{x M_i} \right)^{x M_i-1} \approx \frac{x}{exp(x)}
\]

Figure 5: X-axis [0, 5] fraction of stations to the number of time slots Y-axis: expected cost of successful transmissions
3.4 Provable properties

In current section, we show upper bound on deviation of the experiment from the expected value.

Theorem 1

Proof: The expected number of success is studied in Lemma 1. We concentrate here on proving the tail bound. Similar problem is concerned in [8], but only number of empty time (null) slots is concerned.

Let \( t \) refer to the ,,virtual” time - the point, at which the first \( t \) stations made the decision in which time slot they will transmit. Let \( \mathbf{F}_t \) be the \( \sigma \)-field generated be the random choice of time slots by the first \( t \) stations.

Let \( S \) be the random variable denoting the number of successful transmissions at time \( t = N \) (after all stations have made their decisions). Let \( Z \) be the random variable denoting the number of ,,empty” time slots (number of nulls).

Let \( S_t \) denote the conditional expectation of \( S \) at time \( t \). Let \( Z_t \) denote the conditional expectation of \( Z \) at time \( t \). The random variables \( S_0, ..., S_N \) form a martingal (\( Z_0, ..., Z_N \) also form a martingal).

Define \( s(Y_s, Y_z, t) \) as the expectation of \( S \) given that \( Y_s \) time slots have exactly one transmission and \( Y_z \) time slots have no transmission at time \( t \).

Similarly, define \( z(Y_z, t) \) as the expectation of \( Z \) given that \( Y_z \) time slots have no transmission at time \( t \). As we will see later, \( z \) depends only on \( Y_z \) and \( t \), and \( s \) depends not only on \( Y_s \) and \( t \), but also on \( Y_z \).

Let us consider following series: \( s_i \) and \( z_i \) equal to the expected value of number of success.

\[
s_i = E[S|\mathbf{F}_i]
\]
\[
Z_i = E[Z|\mathbf{F}_i]
\]

The random variables \( S_0, ..., S_N \) form a martingal (\( Z_0, ..., Z_N \) also form a martingal).

Define \( s(Y_s, Y_z, t) \) as the expectation of \( S \) given that \( Y_s \) time slots have exactly one transmission and \( Y_z \) time slots have no transmission at time \( t \).

Similarly, define \( z(Y_z, t) \) as the expectation of \( Z \) given that \( Y_z \) time slots have no transmission at time \( t \). As we will see later, \( z \) depends only on \( Y_z \) and \( t \), and \( s \) depends not only on \( Y_s \) and \( t \), but also on \( Y_z \).

Let us consider following series: \( s_i \) and \( z_i \) equal to the expected value of number of success.

\[
E[s_i+1] = \begin{cases} 
    s_i \quad \text{with probability} \quad 1 - \frac{z_i + s_i}{M} \\
    s_i + 1 \quad \text{with probability} \quad \frac{z_i}{M} \\
    s_i - 1 \quad \text{with probability} \quad \frac{s_i}{M}
\end{cases}
\]

So, \( E[s_{i+1}] = \left(1 - \frac{1}{M}\right) s_i + \frac{1}{M} z_i.\)
\[ z_{i+1} = \begin{cases} \frac{1}{2} & \text{with probability} \ 1 - \frac{1}{2}M \\ z_i & \text{with probability} \ \frac{1}{2}M \end{cases} \]

So, \( E[z_{i+1}] = (1 - \frac{1}{M}) z_i \).

Let \( s(x) = \sum_{i=0}^{\infty} s_i x^i \) and \( z(x) = \sum_{i=0}^{\infty} z_i x^i \).

\[
z(x) = \sum_{i=0}^{\infty} z_i x^i = Y_z + \sum_{i=1}^{\infty} z_i x^i = Y_z + \sum_{i=0}^{\infty} z_{i+1} x^{i+1} =
\]

\[
= Y_z + x \left( 1 - \frac{1}{M} \right) \sum_{i=0}^{\infty} z_i x^i = Y_z + x \left( 1 - \frac{1}{M} \right) z(x)
\]

So, \( z(x) = \frac{Y_z}{1 - (1 - \frac{1}{M}) x} \), \( z_i = Y_z (1 - \frac{1}{M})^i \) and finally:

\[
Z_t = Y_z \left( 1 - \frac{1}{M} \right)^{n-t}
\]

\[
s(x) = \sum_{i=0}^{\infty} s_i x^i = Y_s + \sum_{i=1}^{\infty} s_i x^i = Y_s + \sum_{i=0}^{\infty} s_{i+1} x^{i+1} =
\]

\[
= Y_s + x \sum_{i=0}^{\infty} x^i \left[ (1 - \frac{1}{M}) s_i + \frac{1}{M} z_i \right] = Y_s + \left( 1 - \frac{1}{M} \right) x s(x) + \frac{x}{M} z(x)
\]

So,

\[
s(x) = \frac{Y_s}{1 - (1 - \frac{1}{M}) x} + \frac{x Y_z}{M (1 - (1 - \frac{1}{M}) x)^2}
\]

Thanks to:

\[
\sum_{i \geq 0} \binom{k+i}{k} x^i = \frac{1}{(1-x)^{i+1}}
\]

we get:

\[
s_i = \left( 1 - \frac{1}{M} \right)^{i-1} \left[ Y_s \left( 1 - \frac{1}{M} \right) + \frac{Y_z}{M} i \right]
\]

And finally:

\[
S_t = \left( 1 - \frac{1}{M} \right)^{n-t-1} \left[ Y_s \left( 1 - \frac{1}{M} \right) + \frac{Y_z}{M} (n-t) \right]
\]

To apply the Azuma’s inequality, we have to bound differences \( c_t = S_t - S_{t-1} \).
With probability $1 - \frac{Y_{z,t-1} + Y_{s,t-1}}{M}$ we have:

$$\delta_t = S_t - S_{t-1} = s(Y_{z,t}, Y_{s,t}, t) - s(Y_{z,t-1}, Y_{s,t-1}, t - 1) =$$

$$= s(Y_{z,t-1}, Y_{s,t-1}, t) - s(Y_{z,t-1}, Y_{s,t-1}, t - 1) =$$

$$= \left(1 - \frac{1}{M}\right)^{n-t-1} \left[ Y_{s,t-1} \left(1 - \frac{1}{M}\right) + \frac{Y_{z,t-1}}{M} (n - t) \right] -$$

$$- \left(1 - \frac{1}{M}\right)^{n-t} \left[ Y_{s,t-1} \left(1 - \frac{1}{M}\right) + \frac{Y_{z,t-1}}{M} (n - t + 1) \right] =$$

$$= \frac{1}{M} \left(1 - \frac{1}{M}\right)^{n-t-1} \left[ \left(1 - \frac{1}{M}\right)(Y_{s,t-1} - Y_{z,t-1}) + \frac{1}{M} (n - t)Y_{z,t-1} \right]$$

With probability $\frac{Y_{z,t-1}}{M}$ we have:

$$\delta_t = S_t - S_{t-1} = s(Y_{z,t}, Y_{s,t}, t) - s(Y_{z,t-1}, Y_{s,t-1}, t - 1) =$$

$$= s(Y_{z,t-1} - 1, Y_{s,t-1} + 1, t) - s(Y_{z,t-1}, Y_{s,t-1}, t - 1) =$$

$$= \left(1 - \frac{1}{M}\right)^{n-t-1} \left[ (Y_{s,t-1} + 1) \left(1 - \frac{1}{M}\right) + \frac{Y_{z,t-1} - 1}{M} (n - t) \right] -$$

$$- \left(1 - \frac{1}{M}\right)^{n-t} \left[ Y_{s,t-1} \left(1 - \frac{1}{M}\right) + \frac{Y_{z,t-1}}{M} (n - t + 1) \right] =$$

$$= \left(1 - \frac{1}{M}\right)^{n-t-1} \left[ \frac{1}{M} \left(1 - \frac{1}{M}\right)(Y_{s,t-1} + Y_{z,t-1}) + \frac{1}{M} (n - t) \left(\frac{Y_{z,t-1}}{M} - 1\right) + \left(1 - \frac{1}{M}\right) \right]$$

With probability $\frac{Y_{s,t-1}}{M}$ we have:

$$\delta_t = S_t - S_{t-1} = s(Y_{z,t}, Y_{s,t}, t) - s(Y_{z,t-1}, Y_{s,t-1}, t - 1) =$$

$$= s(Y_{z,t-1}, Y_{s,t-1} - 1, t) - s(Y_{z,t-1}, Y_{s,t-1}, t - 1) =$$

$$= \left(1 - \frac{1}{M}\right)^{n-t-1} \left[ (Y_{s,t-1} - 1) \left(1 - \frac{1}{M}\right) + \frac{Y_{z,t-1}}{M} (n - t) \right] =$$
\[- \left(1 - \frac{1}{M}\right)^{n-t} \left[ Y_{s,t-1} \left(1 - \frac{1}{M}\right) + \frac{Y_{z,t-1}}{M} (n-t+1) \right] =
\]

\[= \left(1 - \frac{1}{M}\right)^{n-t-1} \left[ \frac{1}{M} \left(1 - \frac{1}{M}\right) (Y_{s,t-1} - Y_{z,t-1}) + \frac{Y_{z,t-1}}{M^2} (n-t) - \left(1 - \frac{1}{M}\right) \right] \]

...  

... (może bitę mocniej), na razie jest tak:

\[|c_t| \leq (1 - \frac{1}{M})^{n-t-1} \]

\[
\sum_{t=1}^{n} \bar{c}_t^2 = \frac{1 - (1 - \frac{1}{M})^{2(n-2)}}{1 - (1 - \frac{1}{M})^2} = \frac{M}{N^2} \frac{N^2 - \mu^2 (1 - \frac{1}{M})^{-2}}{2M - 1}
\]

For \(M = N\) we get:

\[P[|S_N - S_0| \geq \lambda] \leq 2exp \left( - \frac{\lambda^2}{2 \sum_{k=1}^{N} \bar{c}_k^2} \right) = 2exp \left( - \frac{\lambda^2 N^2 (2M-1)}{2M (N^2 - \mu^2)} \right) =
\]

\[= 2exp \left( - \frac{\lambda^2 N^2 (2M-1)}{2MN^2 (1 - (1 - \frac{1}{M})^{2(N-1)} (1 - \frac{1}{M})^{-2})} \right) \approx 2exp \left( - \frac{9}{8} \lambda^2 \right)
\]

\[\Box\]

### 3.5 Distribution of singles

In this section we will find the distribution of singles. In fact, we will only find recursive equation.

To find the distribution, we have to thing about the algorithm in another way. Let’s think, that the stations does not decide in which time slot they will transmit in advance, but they make they decision on-line. So, every station instead of choosing time slot with probability \(\frac{1}{M}\) at the beginning of a round, it transmits ...

Then, the probability, that in the \(i\)-th time slot, \(k\) stations will transmit, under assumption, that there was \(l\) stations transmitting in previous \((1, \ldots, i-1)\) time slots is equal to:

\[P_{l,k}^{i} = \binom{N-l}{k} \left(\frac{1}{M-i+1}\right)^k \left(1 - \frac{1}{M-i+l}\right)^{N-l-k} \]
Now, we define variables $B_{i,k}^l$ equal to the probability that after $i$ time slots, $l$ stations have been transmitting and $k$ of them succeeded. We start from point $B_{0,0}^l = 1$ and we are interested in finding values of variables $B_{N,k}^M$ for $k = 0, ..., \min(M, N)$.

$$B_{i+1,k}^l = B_{i,k}^l - B_{i-1,k}^l P_{i-1,1}^l + \sum_{m=0, m \neq 1}^l B_{i-m,k}^m P_{i-m,m}^l$$

Source code of the program written in java which finds the distribution is in appendix.

Based on that distribution, we can easily find following conditional distribution:

It seems to be not proper to approximate $(1 - 1/M_i)^{N_i-1}$ by $exp(-N_i/M_i)$, but in fact that approach is good for values used in our system ($M > 50$).

On the figure below, we can see how expected ratio of successful transmissions depends on ratio between number of stations taking part in the experiment to the number of available time slots.

In BSAA we concern only case, when $\frac{N}{M} \leq 1$, but in further modifications we allow that ratio is greater than 1.

**Theorem 2** For Basic Size Approximation Algorithm with number of time slots equal to $N_{\text{max}}$ in each round ($M_i = N_{\text{max}}$), expected approximation ratio achieved within 3 rounds is 1.1 and 1.01 within 4 rounds.

**Proof:** Let $N_i$ denotes number of active stations in i-th round. Let $c_i = \frac{N_i}{M_i}$ be expected ratio of number of active stations to the number of time slots. Assuming that number of time slots is the same in each round ($M_i = M$ for all $i$), we get: $c_i = \frac{N_i}{M}$. Using Lemma 1 we can write down following recursive expression: $N_{i+1} = N_i(1 - 1/M)^{N_i-1}$, so we can approximate $c_i$ by (for sufficiently large M):

$$c_{i+1} = c_i(1 - e^{-c_i})$$

$c_i$ is the expected ratio of active stations to the number of time slots in i-th round. Number of active stations in i-th round is equal to the number of those stations, which did not have successful transmission in previous rounds ($1, ..., i - 1$). That means that ratio of counted stations (good transmissions) at the beginning of the round $i$ is equal to $1 - c_i$. So, at the beginning of the round $i$, expected approximation ratio is equal to $1 - c_i$.

Calculating first values of $c_i$ we get:

$$c_1 = 1$$
\[ c_2 = 0.63212 \]
\[ c_3 = 0.29617 \]
\[ c_4 = 0.075920 \]
\[ c_5 = 0.0055505 \]

Which means that after 3 rounds expected approximation ratio is equal to 0.92408 and after 4 rounds to 0.99445.

In Leader Election Algorithm from following sections we do not need to be so precise. As we will see in next section, we can err even more than twice (approximation ratio 2) without losing required properties on speed and reliability of leader election.

DOKLADNY ROZKLAD
One can find distribution
KONIEC DOKLADNY ROZKLAD

3.6 One Round BSAA

BSAA achieves good results for approximation, but has running time, which we cannot accept in our settings. We want to approximate number of nodes faster. In BSAA nodes were waiting until all of them succeed with their transmissions. In one round SA we will omit further round. We will estimate size of the network on the number of successful transmissions in first round. In that approach we loose some accuracy, but we gain time.

As it was shown in Theorem 2 expected ratio of number of successful transmissions to the number of time slots in the worst case (the number of time slots is equal to the number of stations in the sector) in one round is 0.36788. So, stations have to transform somehow the results of the experiment, to achieve good approximation of the number of stations in the sector.

In current section we are analyzing only one round of the algorithm, so let us use \( Z, M, N \) instead of \( Z_i, M_i, N_i \) respectively.

3.6.1 Experimental results

We are strongly interested in finding following conditional distribution - \( P(N = n | Z = i) \) - probability that there is exactly \( n \) stations and there was \( i \) success. To set that we have to calculate following sums, for every \( i = 1, ..., N_{max} \):

\[
P(N = n | Z = i) = \frac{\binom{n}{i} p^i (1 - p)^{n-i}}{\sum_{n=1}^{N_{max}} \binom{n}{i} p^i (1 - p)^{n-i}} \tag{8}
\]
We made 1.5 billion of independent SAA experiments, for maximum number of time slots equal 150. Number of stations was random variable from uniform distribution \((N \in [1, 150])\). Let’s denote by \(Z^{i}\) number of success in \(i\)-th probe. Based on experimental data, we calculated following conditional distribution: \(P(N = n|Z = k)\), based on this, for every experiment we calculated absolute error of estimation:

\[
err(i, n) = \frac{|h(i) - n|}{n}
\]

for function \(h\) which, given on the input the number of successful transmissions, estimates number of stations in the sector (in fact, function \(h\) is different from \(g\)). \(err(i, n)\) is the absolute error which every station makes, during estimation, when it uses function \(h\) for that.

Right now our goal is to minimize following sum (by finding right function \(h\)).

\[
ERR(i) = \sum_{n=1}^{150} err(i, n) \cdot P(N = n|Z = i)
\]

Setting proper \(h\) lead us to following bound:

\[
\forall i \in [0, 150] \quad ERR(i) \leq 0.2
\]

So, expected approximation ratio we get is smaller than 1.2.

Function \(h\) was found based on the experiments, where the number of stations was from uniform distribution. It is intuitively clear that number of cars on the highway has different distribution than uniform. But testing the system in case of uniform distribution, gives us Independence from level of traffic.

### 3.7 Third approach

Our last algorithm is joining ideas from previous two. We run more than one round (like in the first one), but we does not demand number of time slots in round to be greater than maximum number of stations. So now we allow that \(c = \frac{N}{M} > 1\). As we can see on Figure 3.5 expected value of \(Z\) is no longer injection. So we cannot the use same techniques as before to prove properties of the algorithm. We show (based on experimental results) that four round-algorithm, which has running time equal to \(2 \cdot M\) slots has very good properties.

First of all, for \(c \leq 1/3\) stations do not need to calculate any estimators. Probability, that observed number of success is equal to the number of stations is greater than 0.9.
For number of success greater than one-third of maximum number of stations we construct function, which minimize absolute error function defined in equation 10.

3.8 Remark about collision detection model

In no-CD model, stations can tell only if there was a successful transmission or not. They cannot differentiate between many transmissions and the empty channel. Using separated transmitter and receiver we achieved that every station, which had failed transmission knows about it, so it can determine its behavior based on that event (stay active/become inactive).

In CD model stations know much more. They can distinguish no transmission and collision. So during SAA they observe not only number of successful transmissions, but also number of collisions. So, they can approximate number of stations by taking the sum of transmissions succeed and double number of failures.

Due to cost of devices, which are able to detect collisions, we do not consider that model, but we want to remark that better results are possible to achieve when collision detection is possible.

3.9 Experimental comparison

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>BSAA</th>
<th>ESAA</th>
<th>BNC(cd)</th>
<th>BSAA</th>
<th>ESAA</th>
</tr>
</thead>
<tbody>
<tr>
<td>running time</td>
<td>150</td>
<td>150</td>
<td>150</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>number of time slots(^1)</td>
<td>150</td>
<td>75</td>
<td>150</td>
<td>150</td>
<td>75</td>
</tr>
<tr>
<td>number of rounds</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>confidence interval(^2)</td>
<td>4</td>
<td>13</td>
<td>25</td>
<td>25</td>
<td>57</td>
</tr>
<tr>
<td>max. likelihood reliability est.(^3)</td>
<td>17</td>
<td>40</td>
<td>56</td>
<td>61</td>
<td>102</td>
</tr>
<tr>
<td>max. num. of success(^4)</td>
<td>81</td>
<td>85</td>
<td>145</td>
<td>135</td>
<td>141</td>
</tr>
</tbody>
</table>

\(^1\) per round

\(^2\) one point confidence interval with confidence ratio 0.9 for plain data (without estimation), maximum \(n\) that \(P(N = n|Z = n) > 0.9\)

\(^3\) maximum \(n\) that \(P(N = n|Z = n) \geq P(N = k|Z = n)\)

\(^4\) maximum number of success during 1.5 million experiments

Let us explain values from table above. Running time is equal to the number of time slots reserved for the algorithm, it is equal to the number of time slots per round multiplied by the number of rounds. We consider three parameters as a determinants of the quality of solutions. First is the confidence interval, more detailed, we find such maximum \(n\) form which \(P(N = n|Z = n) > 0.9\)...

Maximum likelihood reliability estimator is such \(n\) that ...
3.10 Link unreliability

In proofs of Lemma 1 and Theorem 2, we omitted problem of link unreliability to avoid complex analysis. The values we have proved does not change to much when we take into account that problem.

In case of one-round size approximation algorithms: When we assume that every station in the sector, hears every transmission with probability $p_r$, instead of estimating with ratio $c$ it approximates with ratio $p_r \cdot c$ ($p_r \cdot cN \geq n$).

In case of multi round size approximation algorithms analysis is more complex, but gives the same level of approximation loss.

4 Leader election

4.1 Assumptions

In this section we introduce basic algorithm of leader election (LEA for short). We prove its properties, which depend on quality of size approximation algorithm used.

We assume that the number of nodes trying to elect a leader is equal to $N$, but those nodes know only approximation of that value, $n$. Later, we will show performance of algorithm as a function of $n$ (for $n \in [N/5, 3N]$), or which is more handy, as a function of variable $x = n/N$, in fact equal to the size approximation ratio.

4.2 Basic Algorithm

Algorithm runs in steps. During single step nodes transmit „single” bit messages with probability equal to $1/n$. First node, which has successful transmission becomes a leader. For next time-slots reserved for Leader Election Algorithm, leader sends messages to preserve „self-election” by other node, which did not hear his electing message.

Algorithm Properties Probability of success in single transmission (election of leader in one step) is equal:

$$p = \left(\begin{array}{c} N \\ i \end{array}\right) \frac{1}{n} \left(1 - \frac{1}{n}\right)^{N-1}$$

(12)

Let $X$ denote a random variable equal to the number of steps of the algorithm needed to elect the leader, then:

$$P(X = k) = (1 - p)^{k-1}p$$

(13)
Thus, $X$ is the random variable from geometric distribution.

So, expected running time of LE is:

$$E(X) = \sum_{k=0}^{\infty} kP(X = k) = \frac{1}{p}$$  \hspace{1cm} (14)

We are strongly interested in reliability of our protocol, we want to achieve high probability that LEA ends successful.

We can estimate the probability that leader election fail in $k$ steps.

$$P(X > k) = 1 - P(X \leq k) = 1 - \sum_{i=1}^{k} (1 - p)^i p = (1 - p)^k$$  \hspace{1cm} (15)

### 4.3 Experimental results

On Figures 6 & 7 we can see functions, of approximation error. On the first 4.2, we have dependence between approximation error of number of stations and time needed to successfully choose the leader. On the second 4.2 we can see probability that the leader will not be elected within 20 steps.

With assumption that we do not underestimate more than twice, and overestimates less than three times, we reserve 20 time-slots for each leader election to achieve probability smaller than 0.01.

### 4.4 Alternative approach

Now, we can see that we do not have to use any algorithm which estimates number of stations, thanks to the fact that we can underestimate number of nodes $B_{bottom} = 2$-times and overestimate by 3 ($= B_{upper}$). We can think about following pseudo-SAA procedure: Because we have upper bound on maximum number of nodes in the sector $N_{max} = 150$, we can say (in most cases) that number of stations is equal to $\frac{N_{max}}{B_{upper}}$ ($= 50$) for number of nodes greater than $\frac{N_{max}}{B_{bottom} \cdot B_{upper}}$ ($N \geq 25$). In that case, we do not overestimate or underestimate number of stations more than we are allowed to. The same reasoning we can apply for the number of stations smaller than $\frac{N_{max}}{B_{bottom} \cdot B_{upper}}$. That leads us to the following values: for number of stations between 1, ..., 6 the allowed estimation is equal to 3, for number of stations between 6, ..., 36 - 18. The only problem we have to solve is to choose proper estimation (3, 18 or 50). But we can do as follows: make leader election with one of the values, if leader was chosen than we use that estimation for next leader elections until it fails. Than we chose another value (we choose that values periodically: 3 → 18 → 50 → 3 → ...).
4.5 Link unreliability

Let us remaind that $p_r$ denotes the probability that station receive transmission successfully and $N$ is the number of stations in current sector and $n$ is the estimation of $N$, known by each station.

We have to consider following cases:

- station A did not hear ,,leader electing message” (LEM) other stations, which heard it, in next time slots transmits its messages, creating noise. Probability, that station A does not hear that messages (and detect noise) is small ($err_1(N) \approx (1 - p_r)^{p_r N}$). $err$ is decreasing function of $N$, for $p_r = 0.9$ we get $err(2) = 0.1$, $err(3) = 0.01$, ...

- two stations: A and B ,,think”, that they became a leader: it means that A have transmitted LEM and did not hear LEM sent by B (and B did not hear LEM sent by A). The probability, that A and B sent their leader electing messages in the same time slot and did not hear (both of them) NOISE is equal to $err_2(n) = p_r^2 \frac{1}{n^2}$. We get $err_2(n) \leq 0.01$ for $n \geq 9$.

4.6 Consequences of link unreliability

In previous chapter, we have shown, that our Size Approximation Algorithm with high probability errs no more than $f()$. The probability used by single station in LEA in each step is equal to $p_i = 1/f()$. So one can think about the probability of success in single step as a function of $n$ variables: $p_1, ..., p_n$ and is equal to:

So, the next couple of pages we will investigate function:

$$f(x_1, ..., x_n) = \sum_{i=1}^{n} x_i \Pi_{k \neq i} (1 - x_k) = \sum_{i=1}^{n} \frac{x_i}{1 - x_i} \Pi_{i=1}^{n} (1 - x_i)$$

We are interested in behavior of $f$ around point $(1/n, ..., 1/n)$. We will show that it is saddle point of $f$, for $N > 3$.

$$\frac{d}{dx_i} f(x_1, ..., x_n) = \prod_{k \neq i} (1 - x_k) \left[ 1 - \sum_{k \neq i} \frac{x_k}{1 - x_k} \right]$$

Intuition sais, that extreme point of $f$ is $(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n})$. That is true, dfferate of $f$ is equal to 0 in that point:
f(\frac{1}{n}, ..., \frac{1}{n}) = \\
= (1 - \frac{1}{n})^{n-1} \left[ 1 - (n-1)\frac{\frac{1}{n}}{1-\frac{1}{n}} \right] = \\
= (1 - \frac{1}{n})^{n-1} \left[ 1 - (n-1)\frac{n}{(n-1)n} \right] = 0

Now, we will investigate, what kind of extreme point it is. Thanks to the fact that partial differentials of f are in C^1(0,1) (continuous, differentiable functions over (0,1) interval).

\[
\frac{d^2}{d^2x_i, d^2x_j} f = \prod_{k \neq i,j} (1 - x_k) \left[ 1 - \sum_{k \neq i,j} \frac{x_k}{1 - x_k} \right], \text{for} \ i \neq j
\]

and \ \frac{d^2}{d^2x_i, d^2x_i} f = 0.

We will calculate the values of minors of the hessian of f in point \( A = (\frac{1}{n}, ..., \frac{1}{n}) \).

\[
\frac{d^2}{d^2x_i, d^2x_j} f(A) = - \left( \frac{n-1}{n} \right)^{n-3}
\]

\[
M_1(A) = 0
\]

\[
M_2(A) = \text{det} \left( \begin{array}{cc}
0 & f_{1,2}(A) \\
f_{2,1}(A) & 0
\end{array} \right) = f_{1,2}^2(A) > 0
\]

\[
M_3(A) = \text{det} \left( \begin{array}{ccc}
0 & f_{1,2}(A) & f_{1,3}(A) \\
f_{2,1}(A) & 0 & f_{2,3}(A) \\
f_{3,1}(A) & f_{3,2}(A) & 0
\end{array} \right) = 2f_{1,2}^3(A) < 0
\]

\[
M_4(A) = \text{det} \left( \begin{array}{cccc}
0 & f_{1,2}(A) & f_{1,3}(A) & f_{1,4}(A) \\
f_{2,1}(A) & 0 & f_{2,3}(A) & f_{2,4}(A) \\
f_{3,1}(A) & f_{3,2}(A) & 0 & f_{3,4}(A) \\
f_{4,1}(A) & f_{4,2}(A) & f_{4,3}(A) & 0
\end{array} \right) = -3f_{1,2}^4(A) < 0
\]

The hessian of second differential of f is indefinite, which means that f has saddle point in A. It means, that in some cases, probability of success in single step in leader election algorithm can be even higher when stations do not have precise estimation of the network size than in the case, when every station knows exactly the number of number of stations. Thanks to that
feature, the link unreliability scenario seems to be much more interesting than homogenous scenario.

Of course it can happen that the probability is much lower, even if only some small group of the stations err. In upcoming calculus we show bound on the variation of the probability (in the neighbourhood of point $A$).

5 Broadcast

Within this section we describe the broadcasting algorithm. The description is divided into a presentation of the model, a detailed description of the algorithm and an evaluation of its properties.

The evaluation is done in two steps – first we describe the behaviour of the algorithm within a simplified setting, then we introduce a full description. The simplified environment assumes that a message is the only one in the system, so that it does not end up in any queues and is not disturbed by other messages. Its description is given because it provides an easier introduction for the reader and because such a situation occurs at the very most critical point of time in the system – with the first notification of danger travelling through the system. In the second, general setting, we evaluate the possible interactions between different messages.

The presentation of each of these settings ends with an evaluation of experimental results.

5.0.1 Model

Recall that we have created a framework consisting of g-slots and gs-slots. A leader is only permitted to transmit a message in the gs-slot which is fixed for its sector. This gs-slot must be contained in a g-slot for $BCAST$. Within each gs-slot only one message can be transmitted. The division of time in slots is described in section 2.3.3. For clarity, g-slots for $BCAST$ are called bg-slots within the next part of the paper.

For each transmission slot, there is (with very high probability) exactly one leader elected in each sector, elected in the last g-slot for $LEA$. Only this leader is active within this transmission slot, all other stations are participating passively.

The sectors are uniquely numbered in driving direction of the stations by increasing integers. The distance between two sectors numbered $S$ and $S'$ is defined as $d(S, S') = S - S'$. The distance of neighbour sectors is always 1. The term “previous sector” describes the sector which lies ahead in driving
direction and has an ID smaller by 1. Lying ahead in driving direction means also that all new messages come from the previous sector.

We assume that the stations are travelling at equal distances so that there are $n$ nodes in each sector. Assuming a constant speed $v_n$ of each station, within each second $l_s := (n/s_l)v_n$ nodes leave the sector at one side, and exactly the same number of stations joins the sector at the opposite side. We assume that $n$ is sufficiently large for the considerations in this section, so that the analysis is not applicable if $n$ is small. Section 6 deals with the case that $n$ is small.

5.0.2 Algorithm

Let us start by enumerating environmental properties and parameters essential for the algorithm. These are determined by the available bandwidth, transmission range and the performance of the size approximation and leader election algorithms. Table 1 describes the properties.

**Message generation** When a message is generated, it obtains a unique ID.

**Station state** Each station remembers all possible messages, identifying them by their ID. With each message it associates a state class and a counter. The counter is used for counting the number of heard transmissions (which can be lower than the number of actually sent transmissions) containing this particular message and sent from the current sector. The initial state class of each message is IDLE. The initial value of each counter is zero. The state class of message $M$ for station $s$ is denoted by $\text{state}(s, M)$. The counter is denoted by $\text{counter}(s, M)$.

The state class for one message ID can be obviously different for different stations – the state class describes for a station what it knows about a particular message ID.

Additionally, each station maintains a FIFO queue. In this FIFO queue it stores message IDs for messages which have state NEW (for this station). Every time a message changes its state class from IDLE to NEW it is appended to the queue, when it gets state class FINISHED it is removed from the queue. Initially the queue is empty for each station. We refer to the first element of the queue as the “primary message” of the station and denote it by $\text{primary}(s)$.

The various state classes which are associated by stations with messages are:
Table 1: Parameters for the broadcasting algorithm

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{mb}$</td>
<td>defines the amount of time which passes between two the beginnings of two gs-slots for BCAST for one sector (in seconds). This parameter is defined by the way the communication slots are assigned for particular algorithms and for particular sectors.</td>
</tr>
<tr>
<td>$s_l$</td>
<td>sector length, set to $r_{max}/2$ (recall that $r_{max}$ describes the maximum transmission range). This setting implies that two stations in neighbour sectors always lie within transmission range.</td>
</tr>
<tr>
<td>$p_r$</td>
<td>the probability that a stations will receive a proper transmission successfully.</td>
</tr>
<tr>
<td>$r_b$</td>
<td>defines the number of repetitive transmission of one message $M$ that should be executed within each sector.</td>
</tr>
<tr>
<td>$d_{max}$</td>
<td>it is not necessary to inform stations which are too far away about an event. Thus $d_{max}$ describes the maximum number of slots a message can be transmitted from its source. This is similar to a maximum TTL (time to live) in TCP/IP networks.</td>
</tr>
<tr>
<td>$t_r$</td>
<td>a message cannot stay longer then this ready for broadcasting at a station.</td>
</tr>
<tr>
<td>$t_g$</td>
<td>the minimum time between new generated messages.</td>
</tr>
<tr>
<td>$N$</td>
<td>the maximum number of nodes in a sector.</td>
</tr>
</tbody>
</table>

**IDLE** never heard about this message,

**NEW** heard about this message from the previous sector, it should be transmitted to the next sector as soon as possible,

**FINISHED** there have been some processing for this message, now we forget about it and ignore any transmissions for this message.

**Transitions** The following state transitions for station $s$ describe the algorithm:

- whenever a transmission with message $M$ is heard from the preceding sector and state($s, M$) = IDLE, the state class changes to NEW (and it is automatically appended to the FIFO queue),
• every time a transmission with message $M$ is heard from the current sector, the transmission counter $\text{counter}(s, M)$ for message $M$ is increased,

• when the counter of transmissions for a message $M$ reaches $\text{counter}(s, M) \geq r_b$, the state class of $M$ is changed to FINISHED (and the message is removed from the FIFO queue),

• if $\text{primary}(s) = M$ for time longer than $t_r$ (so $M$ is first in FIFO queue for longer than the timeout), $M$’s state class is changed to FINISHED (and it is removed from the FIFO queue),

• when a node changes sectors, the state class of all messages with state class NEW is changed to state class FINISHED (and all these messages are removed from the queue).

Moreover, in each transmission slot the current sector leader transmits its primary message, i.e. the message which is first in its FIFO queue. If there is no such message, the leader stays quiet.

A station may be sometimes able to receive a transmission which has been sent by a station located in a sector with distance 2 (not 1 as usual), but this received information is discarded as it is not needed in a scenario with $n$ sufficiently large. We reconsider this issue in 6.

In the following section we want to show that this broadcasting technique gives good results in terms of reliability and speed, assuming that the number of messages is limited and that the number of stations is large enough.

## 5.1 Time-efficiency and reliability in simplified setting

We show bounds on the time-efficiency and the reliability of the broadcasting algorithm, in the case that there is only one message traveling in the system. This assumption is revised later and the considerations about the properties of the broadcasting algorithm are extended for the more general setting.

### 5.1.1 Time-efficiency

In this subsection we will show an upper bound on the time needed for a message to get from its source to another sector. We show that for this task a message needs linear time with respect to the distance between the source and destination sector. Moreover we present a good upper bound on the constants occurring in this lower bound.

The key problems that occur with broadcasting in the considered model are
• if there is a transmission from a sector to the next sector, then normally we would assume that all stations in the next sector receive the transmission. Because of the link unreliability, only a fraction (maybe a majority, but not all) of the stations receives the transmission.

• after one or more transmissions from sector $S$ containing message $M$, there is some number of stations which have knowledge of $M$ (i.e. have received at least one of the transmissions containing $M$) in sector $S-1$. This number decreases with time, as stations knowing about $M$ leave the sector and new stations join the sector.

A possible problem is that a leader can leave a sector when it is transmitting, and thus the transmission might not be completed successfully. We ignore this problem as the amount of time which passes between the election of a leader and the end of the corresponding g-slot for $BCAST$ is small, and is not probable that such a situation will occur.

The evaluation of these problems is done within the next Lemmas and Facts, which are followed by a conclusion providing an upper bound on the time efficiency.

**Lemma 2** The probability that exactly $m$ stations in a sector $S$ get to know about a message $M$ after $x$ transmissions of $M$ originating from sector $S+1$ is

$$P_e(m, x) = \binom{n}{m} (1 - (1 - p_r)^x)^m ((1 - p_r)^x)^{n-m}.$$  

**Proof:** The probability for each station that it receives one transmission is $p_r$ (due to radio channel unreliability). Considering exactly one transmission we have $n$ Bernoulli trials with success probability $p_r$. We can model $x$ transmissions by $n$ Bernoulli trials with probability of success $1 - (1 - p_r)^x$, because $x$ independent transmissions give a probability $(1-p_r)^x$ that one node has not heard any of the transmissions. So we have a binomial distribution for the probability of having exactly $m$ successes in the Bernoulli trial.

The probability for obtaining exactly $m$ successes in the Bernoulli trial with number of experiments $n$ and probability of success $(1 - (1 - p_r)^x)^m$ is given in the thesis.

□

**Lemma 3** If at a transmission slot $x$, at least $k$ stations in a sector $S$ change the state class for message $M$ from IDLE to NEW, then after $t < t_r$ seconds, there at least

$$A(k, t) \geq \max(0, k - l_s t).$$
stations which have message \( M \) with state class NEW, presuming there were less than \( r_b \) transmissions of message \( M \) before \( t \) within sector \( S \).

**Proof:** \( l_s \) stations are leaving the sector each second, and all new stations entering \( S \) do not know anything about this message. So the number of stations which have message \( M \) in state class NEW becomes smaller by at most \( l_s \) stations leaving the sector at each second. As the state class of the broadcast can change only after at least \( r_b \) successful transmissions of the message within the current sector or after the timeout \( t_r \), the stations which still are in the sector still have state class NEW for message \( M \).

\[\square\]

**Lemma 4** We are considering a time point at the beginning of a gs-slot \( s \) for BCAST and for sector \( S \). If there are \( k \) stations in sector \( S \) which have message \( M \) as their primary message, then the probability that message \( M \) will be transmitted in \( s \) is

\[
k/n
\]

**Proof:** The probability distribution of the leader election algorithm for choosing particular stations is uniform. So the leader will be with probability \( k/n \) a station which has \( M \) as its primary message. A leader transmits its primary message whenever it has the possibility, so message \( M \) will be transmitted with probability \( k/n \).

\[\square\]

**Lemma 5** Let there be at least \( k \) stations in sector \( S \) which change the state class of message \( M \) from IDLE to NEW at point of time \( t \). Then the probability that there will be at least one transmissions of message \( M \) in sector \( S \) within the next \( x \leq t_r \) bg-slots is not greater than

\[
P_t(k, x) \geq 1 - \prod_{i=0}^{x} \frac{n - A(k, xt_{mb})}{n}.
\]

**Proof:** The probability that there will be no transmissions of \( M \) in the \( x \)-th bg-slot is at most

\[
\frac{n - A(k, xt_{mb})}{n}.
\]

\( n - A(k, xt_{mb}) \) specifies an upper bound on the number of nodes which do not have message \( M \) as their primary message. Thus by Lemma 4 the probability that there won’t be any transmission is as given in equation (16).
The probability that there won’t be any transmissions in $x$ slots is then
\[
\prod_{i=0}^{x} \frac{n - A(k, xt_{mb})}{n}
\]
and thus the probability that there will be at least one transmission is the complementary probability.

The definition of $P_t$ allows us to define with how much probability there will be at least one transmission of message $M$ in a sector $S$. Let us assume there is a point of time $t$ when the leader of sector $S + 1$ transmits for the first time the message $M$. Then, with probability $P_t(k, 1)$ there exactly $k$ stations which have successfully received this message in sector $S$.

Basing on the assumption that there are at least $k$ stations in $S$ which have received $M$, we can say that with probability $P_t(k, x)$ the message will be transmitted at least once in $x$ bg-slots. The event “there are $k$ stations in $S$ which have received $M$ and there was at least one retransmission of $M$ within the next $x$ g-slots” occurs with probability $P_c(k, 1)P_t(k, x)$.

As these events are independent, we can sum them and obtain
\[
\sum_{k=1}^{n} P_c(k, 1)P_t(k, x),
\]
which defines the probability that there will be at least one transmission of $M$ in sector $S$ in $x$ bg-slots after there have been at least one transmission of $M$ from sector $S + 1$.

We can evaluate the sum (17) for different $x$ values and so find such a number of bg-slots that it is very probable that the message $M$ will be transmitted in this time in sector $S$.

Actually we want to give an upper bound on the time needed for message $M$ to get from a sector $S$ to a sector $S'$. When the message is passed with high probability (given in eq. (17)) between two neighbour sectors within at most $x$ bg-slots, then the number of bg-slots which are needed for message $M$ to get from $S$ to $S'$ is $x \cdot d(S, S')$. This time is achieved with probability at least
\[
\left( \sum_{k=1}^{n} (P_c(k, 1)P_t(k, x)) \right)^{d(S, S')}
\]

**Parameters** The values of $n$, $p_r$ and $v_n$ have an influence on the considerations about time-efficiency we have made. We will show the performance of the algorithm as a function of all of these parameters in the next paragraph.
**Experimental results** Within the experiments we assume the following properties:

- \( r_{\text{max}} = 500 \) the maximum transmission range is 500 meters,
- \( s_l = 250 \) the sector length is a consequence of the maximum transmission range,
- \( N = 150 \) the maximum number of nodes is 150,
- \( v_n = 30 \) the speed of stations is 30 m/s, which is about 110 km/h,
- number of bg-slots is 15 for every second.

First we assume an reliability of \( p_r = 0.9 \) and 100 nodes per sector. With these assumptions we try to find such an number of bg-slots that the transmissions of at least one message within these number of bg-slots is very probable. To achieve this, the equation (17) is evaluated for different values of \( x \). Table 2 shows the results.

<table>
<thead>
<tr>
<th>Number of gs-slots</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability for transmission</td>
<td>0.89</td>
<td>0.98</td>
<td>0.998</td>
<td>0.9998</td>
<td>0.99997</td>
<td>0.999997</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Probabilities of at least one transmission of message within given number of gs-slots

We can see that for number of bg-slots equal or greater than 4 the probability is greater then 0.9998. Then, according to equation (18) the probability that the message will reach a sector at distance of 20 sectors within \( 4 \cdot 20 = 80 \) bg-slots is \( 0.9998^{20} = 0.997 \). 80 slots correspond to about 5.3 seconds with the chosen bandwidth.

Let us now look at the number of bg-slots needed for transmission in order to achieve a probability of retransmission higher than 0.99, as a function of \( p_r \) – shown in Figure 10.

Finally we look at the probability of achieving at least one transmission within 4 bg-slots as a function of the number of nodes in one sector, as shown in Figure 11.
5.1.2 Reliability

We define the reliability of the broadcasting algorithm as the probability that a message will reach sector \( S_v \) when originating from sector \( S_1 \).

Consider the sequence of sectors \( S_1, \ldots, S_v \) which are between \( S_1 \) and \( S_v \). If the message \( M \) does not reach sector \( S_v \), then there is some sector \( S_i \) which receives at least one transmission of message \( M \) and does not retransmit it.

The results of the time-efficiency evaluation can be easily reused when considering reliability. Let us look closer at the sector \( S_i \). The probability that this sector won’t retransmit \( M \) is equal to \( 1 - P_t(k, t_r) \), assuming that \( k \) stations in \( S_i \) received \( M \) from \( S_i + 1 \) (recall that \( P_t(k, t) \) defines a probability that there will be at least one transmission of \( M \) within \( t \) slots, when \( k \) stations knew about \( M \)). Applying the same technique as earlier, we can define the sum

\[
\sum_{k=0}^{n} P_c(k, 1)(1 - P_t(k, t_r)). \tag{19}
\]

Equation (19) provides an upper bound on the probability that the message \( M \) won’t be retransmitted in sector \( S_i \), after there was at least one transmission of \( M \) from \( S_i + 1 \). We can easily give a lower bound on the probability that \( M \) will be transmitted in a sector \( S_i \)

\[
\sum_{k=0}^{n} P_c(k, 1)(P_t(k, t_r)). \tag{20}
\]

Thus the probability that message \( M \) will reach \( S_v \) when originating from \( S_1 \) is

\[
\left( \sum_{k=0}^{n} P_c(k, 1)(P_t(k, t_r)) \right)^{d(S_1, S_v)}, \tag{21}
\]

as the message must be successfully retransmitted in each sector between \( S_1 \) and \( S_v \).

**Parameters**  As earlier, the values of \( n \), \( p_r \) and \( v_n \) have an influence on the considerations about reliability. Additionally the maximum distance \( d_{max} \) at which messages are retransmitted is of great value.

**Experimental results**  Within the experiments we assume the following properties:

- \( r_{max} = 500 \) the maximum transmission range is 500 meters,
• $s_t = 250$ the sector length is a consequence of the maximum transmission range,

• $N = 150$ the maximum number of nodes is 150,

• $d_{\text{max}} = 20$ the maximum distance at which messages are transmitted, corresponds to 5 kilometers,

• $t_r = 8$ which is twice the number of g-slots at which the message will be transmitted with high probability, as calculated in the time-efficiency experimental evaluation,

• $v_n = 30$ the speed of stations is 30 m/s, which is about 110 km/h,

• number of bg-slots is 15 for every second.

Let the reliability of the communication links be $p_r = 0.9$. Then the reliability of the broadcasting algorithm as a function of the number of nodes $n$ is shown in Figure 12.

Next we evaluate the reliability for 10 nodes in a sector as function of node speed. Figure 13 shows the reliability at distance 20 with node speed varying from 1 to 60 m/s (where 60 ms/s is about 220 km/h).

5.2 Time-efficiency and reliability in general setting

In the last section we have assumed that there is only one message in the system. Now we turn to a full scenario, where different messages can interact with each other, possibly degrading the performance of the system.

Let us define the propagation speed of a message. We define the speed in terms of sectors per bg-slot. The minimum speed is defined as $v_{\text{min}}$ and the maximum as $v_{\text{max}}$.

The maximum theoretically possible speed is obviously $v_{\text{max}} = 1$. This occurs, when in each possible bg-slot the message is propagated one sector.

The minimum theoretical speed depends on the timeout $t_r$ given in the broadcasting algorithm – if the message isn’t forwarded in $t_r$ transmission slots, then it cannot be forwarded at all (because it loses its NEW state class). Nevertheless the practical $v_{\text{min}}$ can be higher because nodes achieve to transmit the message earlier.

We assume there is only one source of messages and the minimum time between the release of different messages is $t_g$. Even if $t_g > t_r$, the difference between $v_{\text{min}}$ and $v_{\text{max}}$ creates a possibility for growing queues of messages at certain sectors. This is best illustrated if you imagine that one message travels with speed $v_{\text{min}}$ all the time. Then, a message which has been released
\( t_g/t_{mb} \) bg-slots later and travels with speed \( v_{max} \) can catch up with the first message at distance \( d \), where \( dv_{max} + t_g/t_{mb} = dv_{min} \). With \( v_{min} \) denoting the minimum speed given in terms of sectors per bg-slots, we have that \( 1/v_{min} \) describes the number of bg-slots needed to get from one sector to another.

Within our considerations we assume there is only one source of messages, i.e. all messages are generated in one sector.

### 5.2.1 Travelling speed

Let us notice that with very high probability, the message will be transmitted within a finite number \( 1/v_{min} \) of bg-slots \(^1\), if \( v_{min} \) has been defined appropriately. With increasing probability, the slot number is increasing too. The number of slots must be also valid for the situation, when the message is staying in the queues of stations within a sector for some time (defined in the next subsection). Within this time, stations which know about this message escape from the sector, which causes the probabilities for a transmission of this message to decrease.

Let us recall that earlier we have described the probability for achieving at least one transmission of a message within \( x \) g-slots by

\[
\sum_{k=1}^{n} P_c(k,1)P_t(k,x) \tag{22}
\]

This calculation was done with the assumption that the queues at each station are empty, so that the retransmission of the message can proceed right after receiving the message from the previous sector.

Now we must also take into account that the message could wait for some time in the queues of stations waiting for getting the status of a primary message. With a bounded number of messages in the queue \( q_{max}(d) \) we can bound the amount of time a message spends waiting in the queue by \( w(d) = (1/v_{min})q_{max}(d)t_{mb} \). Within the definition of \( q_{max}(d) \) the parameter \( d \) specifies the distance of the current sector from the source. As we will see later, the maximum queue size depends on this parameter.

With a maximum waiting time in the queue \( w(d) \), and \( l_s \) stations leaving the sector within one second, we have that \( w(d)l_s \) stations can leave the sector within \( w(d) \) time. Thus if at a moment \( k \) stations know about the message then after \( w(d) \) time there are at least \( k - w(d)l_s \) stations which know about

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\(^1v_{min}\) is defined in terms of sectors per bg-slot, so its inverse is the amount of bg-slots needed for travelling one sector.
the message. Thus extending equation (22) we have
\[ \sum_{k=1}^{n} P_c(k, 1) P_t(k - w(d)ls, v_{\text{min}}) = \sum_{k=1}^{n} P_c(k, 1) P_t\left(k - \frac{1}{v_{\text{min}}} q_{\text{max}}(d) t_{\text{mb}}ls, x\right) \]

(23)

which specifies the probability that a message will be transmitted at least once from sector \( S \) when there have been a transmission of this message from \( S + 1 \) at most \( w(d) \) time earlier.

Now we turn our attention to the problem of queue size.

5.2.2 Queue size

Recall that the only source of messages generates a new message not earlier than \( t_g \) time after the last message. The generation-rate \( t_g \) is larger than \( t_r \).

**Lemma 6** The maximum queue size at distance \( d \) from source \( S \) is

\[ q_{\text{max}}(d) = \frac{(1/v_{\text{min}} - 1/v_{\text{max}})d + 1/v_{\text{min}}}{t_g} \]

**Proof:** By contradiction let us assume there is some sector \( S_x \) with a station with message queue strictly larger than \( ((1/v_{\text{min}} - 1/v_{\text{max}})d + 1/v_{\text{min}})/t_g \).

The difference in the arrival time between the oldest and most fresh of these messages can be at most \( 1/v_{\text{min}} \) (otherwise, the oldest message would have been already transmitted with great probability and would leave the queue).

Let us consider the time span at which the messages from the queue have been generated by the source \( S \). We are going to find the longest possible time span and show that despite its length it is impossible that more than \( (1/v_{\text{min}} - 1/v_{\text{max}})d + 1/v_{\text{min}} \) messages have been generated within this time.

When the oldest message was travelling with the smallest possible speed, \( 1/v_{\text{min}} \) and the most fresh was travelling as fast as possible, i.e. with speed \( v_{\text{max}} \), then the difference of the generation time of both messages may lay as far as \( (1/v_{\text{min}} - 1/v_{\text{max}})d + 1/v_{\text{min}} \). Of course, there is no possibility that more than \( (1/v_{\text{min}} - 1/v_{\text{max}})d + 1/v_{\text{min}} \)/\( t_g \) messages have been issued by the source at this time, as the interval between messages is not greater than \( t_g \).

As we can see, the definition of the travelling speed depends on the queue size and vice versa. This problem can be solved by providing a first upper bound on the value of one of these parameters and then recursively lowering both values until a constant upper bound is found.
5.2.3 Time-efficiency

The minimum throughput of each sector-to-sector communication link is $1/v_{\text{min}}$. When all messages travel with this minimal speed, there is no interaction between messages. Obviously the time needed for reaching distance $d$ is $v_{\text{min}}d$.

When a message travels faster than $v_{\text{min}}$, then it can catch up with other slower messages at some sector $S_x$. This affects performance but we reason that the overall time needed by a message for travelling does not exceed the bound of $v_{\text{min}}$.

Recall that the time which is needed for a message to be passed to the next sector is defined to be $1/v_{\text{min}}$. This time is defined so that it is obeyed even if the message has been staying in the stations’ queues of the current sector $S_x$ for some constant time. So there is no significant difference whether the message stays in the queue of stations in sector $S_x$ for some time (because it has been travelling faster in the sectors prior to $S_x$) or when it slows down a little in the sectors prior to $S_x$ and does not catch up with older messages in sector $S_x$. But, when it does not reach the older messages, it surely can reach distance $d$ within time $v_{\text{min}}d$.

Experimental results  Within the experiments we assume the following properties:

- $r_{\text{max}} = 500$ the maximum transmission range is 500 meters,
- $s_t = 250$ the sector length is a consequence of the maximum transmission range,
- $N = 150$ the maximum number of nodes is 150,
- $v_n = 30$ the speed of stations is 30 m/s, which is about 110 km/h,
- $t_g = 1$ the time between the generation of two new messages is at least 1 second,
- $p_r = 0.9$ the reliability of the radio communication,
- number of bg-slots is 15 for every second.

For determination of the speed factor, we need to compute a $v_{\text{min}}$, which will be obeyed with great probability. Thus table 3 illustrates the probabilities for transmission of a message within $v_{\text{min}}$ g-slots, depending on the distance $d$ from the source. The computations are performed basing on equation (23) and are performed for 100 nodes in each sector.
The probabilities for 5 g-slots are already all above 0.9995, so we choose this setting. Figure 14 shows the dependency of the probability of transmission within 5 g-slots with the number of nodes in a sector as a parameter, at a distance of 20 sectors.

Let us now look at the queue size for a $v_{\text{min}}$ of 5 g-slots. Figure 15 shows the queue size as a parameter of the distance from the source sector.

### Table 3: Probability of transmission within $v_{\text{min}}$ g-slots depending on the distance from source

<table>
<thead>
<tr>
<th>$v_{\text{min}}$ g-slots</th>
<th>Distance $d$</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.865041</td>
<td>0.864241</td>
<td>0.862907</td>
<td>0.860241</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.979159</td>
<td>0.978478</td>
<td>0.977319</td>
<td>0.974903</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.996396</td>
<td>0.996120</td>
<td>0.995627</td>
<td>0.994514</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.999313</td>
<td>0.999219</td>
<td>0.999040</td>
<td>0.998589</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>0.999857</td>
<td>0.999827</td>
<td>0.999764</td>
<td>0.999582</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.999968</td>
<td>0.999958</td>
<td>0.999936</td>
<td>0.999860</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>0.999992</td>
<td>0.999989</td>
<td>0.999981</td>
<td>0.999948</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>0.999998</td>
<td>0.999997</td>
<td>0.999994</td>
<td>0.999979</td>
</tr>
</tbody>
</table>
Figure 6: On the figure above, we can see the conditional distribution of number of stations for $M = 150$ and for different number of successful transmissions.
Figure 7: X-axis [0, 2] fraction of stations to the number of time slots Y-axis: expected ratio of successful transmissions

Figure 8: X-axis (0.3, 5.0): approximation error on number of stations (output of SAA) Y-axis: number of time slots needed by LEA to complete successfully with high probability
Figure 9: X-axis (0.3, 5.0): approximation error on number of stations (output of SAA) Y-axis: probability that leader election does not succeed in 20 steps

Figure 10: Number of bg-slots needed for transmission. X-axis shows $p_r$ and Y-axis number of bg-slots
Figure 11: Probability of transmission in 4 bg-slots, X-axis shows number of nodes

Figure 12: Reliability of broadcasting, X-axis shows number of nodes
Figure 13: Reliability of broadcasting, X-axis shows node speed

Figure 14: Probability of transmission within 5 g-slots, X-axis shows number of nodes
Figure 15: Maximum queue size, X-axis shows distance from source
5.2.4 Reliability

Recall that we have defined the reliability of the broadcasting algorithm as the probability that a message will reach sector $S_v$ when originating from sector $S_1$.

As earlier we consider the sequence of sectors $S_1, \ldots, S_v$ which are between $S_1$ and $S_v$. If the message $M$ does not reach sector $S_v$, then there is some sector $S_i$ which receives at least one transmission of message $M$ and does not retransmit it.

We can reuse the technique introduced in the evaluation of the time-efficiency within the general setting, which consisted of defining the time $w(d)$ which a message can spend waiting in the queues of stations, determining how many stations can leave the sector within this time and thus modifying to the equation (20). The modified reliability of the algorithm is as follows

$$\left( \sum_{k=0}^{n} P_r(k, 1)(P_t(k - w(d), l_s, t_r)) \right)^{d(S_1, S_v)},$$

where $w(d) = (1/v_{\min})q_{\max}(d)t_{mb}$.

Experimental results  Within the experiments we assume the following properties:

- $r_{\max} = 500$ the maximum transmission range is 500 meters,
- $s_l = 250$ the sector length is a consequence of the maximum transmission range,
- $N = 150$ the maximum number of nodes is 150,
- $v_n = 30$ the speed of stations is 30 m/s, which is about 110 km/h,
- $t_g = 1$ the time between the generation of two new messages is at least 1 second,
- $p_r = 0.9$ the reliability of the radio communication,
- number of bg-slots is 15 for every second.

In Figure 16 we show the reliability as a function of the parameter of reliability of the radio communication $p_r$, at a distance of 20 sectors from the source. The probabilities are very high for all $p_r > 0.4$. 

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Figure 16: Reliability at distance 20, X-axis shows $p_r$. 
6 Modified system for small number of stations

The broadcast algorithm presented above works correctly whenever there are enough stations in each sector, so that whenever a message reaches a sector, there are enough stations for transmitting this message to the next sector before they leave the sector and forget about the message. Problems arise, whenever there are not enough stations for reliably passing messages.

We propose a solution which utilizes the former broadcasting algorithm extended to fulfil the new requirements.

6.0.5 Sector organization

We propose the utilization of two layers of geographical sectors, so that each stations is always in one sector within each layer. The sectors are aligned with a shift of one half of the sector size ($s_l/2$), so that whenever a stations leaves one sector, it is exactly in the middle of the sector in the other layer. Figure 17 presents the two layers and one stations, which is at the same time in sector $x_L + 3$ and $x_R + 4$.

Figure 17: Two layers of sectors
6.0.6 Time organization

The available communication bandwidth is divided by two and each layer obtains its own communication slots. These slots are interleaved, so that each second one-bit a-slot belongs to the left layer. The other half belongs to the right layer. Within its own slots each layer executes the full range of algorithms described in the earlier chapters, thus creating a full broadcasting environment.

6.0.7 Algorithms

Broadcasting is performed parallel in both sector layers. Each message is passed in both the right and left layer, providing some redundancy. More important, every time a stations gets to know about a message it will be in the current sector in one of the layers for at least the time it needs for travelling through half of the sector length.

There is no need for changes within the size approximation and leader election algorithms.

The broadcasting algorithm is modified in order to utilize the information obtained from the broadcasting algorithm running in the other sector layer. Additionally it is modified so that transmissions between stations in distance $s_l$ are always utilized. The performance of the modified algorithm is evaluated for a scenario with a very small number of stations on the road. Afterwards we turn to the previous scenario known for the basic system.

7 Modified broadcasting for small number of stations

The broadcasting algorithm is modified in the following way

- each layer notifies the broadcasting algorithm running in the next layer about each transition from state class IDLE to state class NEW, thus informing about an incoming message,

- as a reaction to notification from the next layer, the broadcasting algorithm sets the state class of the corresponding message from IDLE to NEW, if it is not in state class NEW or FINISHED yet,

- when receiving a message from a sector with distance 2 (lying in the driving direction) for the first time, the state class of this message is changed from IDLE to NEW as if it had been received from a sector with distance 1.
The two first enhancements introduce an interaction between the two parallel running broadcasts, so that all available important information is shared.

The third enhancement provides a possibility for stations travelling in distance $2s_t$ (so $r_{max}$ with the chosen sector length) to exchange information.

Within the rest of this section, we evaluate the speed and the reliability of this scenario for the case when there are stations within the maximum transmission range $r_{max}$, all travelling with equal speed. Later on we come back to the model presented in the earlier sections and show the performance of the modified system within this new model.

### 7.1 Time-efficiency and reliability for small number of stations

Within this subsection we evaluate the performance of the two broadcasting algorithms within the two-layer model. We assume there are stations at exactly $r_{max}$ distance, from the source up to the maximum travelling distance of a message $d_{max}$.

#### 7.1.1 Time-efficiency

Within the current scenario, there is always only one node in a sector and thus the leader choice is obvious. Thus messages are passed always as they are received to the next node, either in both broadcasting layers or only in one, if the node has just left one of the sectors after receiving information about the message.

This means that the speed achieved within a scenario with more nodes is easily achieved with a small number of stations.

#### 7.1.2 Reliability

The reliability within this scenario is worse as with more stations. This is because the only information about a message which a station receives might be $r_b$ transmissions of the message – usually it receives $2r_b$ transmissions from both layers. In a worst case it receives only messages from one layer, as the previous node leaves the sector in one of the layers just after receiving information about the message.

So the probability that the node does not receive information about the message is $(1 - p_r)^{r_b}$ which might be significant. With assumptions of $r_b = 2$ and $p_r = 0.9$ this is $1/100$. 

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7.2 Time-efficiency and reliability with more stations

The speed and reliability of the broadcasting algorithm can be evaluated as within the basic system. We will consider both broadcasting algorithms which are executed parallel in separate. The only interaction between them bases on the notifications about state transitions from IDLE to NEW.

So evaluating the properties of the broadcasting algorithm in the new system, we must consider the following changes to the old broadcasting algorithm:

- the maximum travelling speed of a message can increase up to 2 sectors per transmission slot, as a message received from a sector with distance 2 will also be considered,

- the transition from IDLE to NEW can happen basing on a notification from the next broadcasting algorithm.

7.2.1 Travelling speed and queue size

Fact 1 The maximum travelling speed of the message is $v_{max} = 2$.

Proof: As described above, such a maximum speed can be achieved when each transmission is heard two sectors ahead and is immediately passed to the next sector in distance 2. There is obviously no possibility for a message to travel a distance of $d$ faster than in $d/2$ transmission slots – no matter how the interaction between the two layers of broadcasting is.

Recall that in section 5.2.2 we have defined the maximum queue size $q_{max}$ depending on the maximum speed, the minimum speed and distance. The queue sizes defined can be directly applied for the new broadcasting algorithm. The queue sizes will be higher, as the maximum speed has increased.

$$q_{max}(d) = \frac{(1/v_{min} - 1/v_{max})d + 1/v_{min}}{t_g}$$

Basing on the queue size we can as previously find such a minimum speed that the probability that at least one transmission will occur during this time is high enough. The previously defined probability $P_l$ can be used without any restrictions for this purpose.
Experimental evaluation  Within the experiments we assume the follow-
ing properties:

- \( r_{\text{max}} = 500 \) the maximum transmission range is 500 meters,
- \( s_l = 250 \) the sector length is a consequence of the maximum transmis-
sion range,
- \( N = 150 \) the maximum number of nodes is 150,
- \( v_n = 30 \) the speed of stations is 30 m/s, which is about 110 km/h,
- \( t_g = 1 \) the time between the generation of two new messages is at least
  1 second,
- \( p_r = 0.9 \) the reliability of the radio communication,
- number of bg-slots is 15 for every second.

First we give a comparison of the queue sizes, as opposed to the values
given in Figure 15. Figure 18 shows the maximum queue size as a parameter
of distance from the source, with \( v_{\text{min}} \) equal to 5.

Figure 18: Maximum queue size, X-axis shows distance from source
7.2.2 Reliability

The reliability of the modified system is much better than that of the previous system. This is because both broadcasting algorithms must fail to pass a certain message within a short distance at the same time. If only one broadcasting algorithm fails to pass the message, the next layer will pass it on and the first layer will get notifications from the next layer, thus bypassing the fault.

Recall that the probability that a message gets lost between sector $S_1$ and $S_v$ within one broadcasting algorithm is defined by

$$1 - \left( \sum_{k=0}^{n} P_c(k, 1)(P_t(k - w(d)l_s, t_r)) \right)^{d(S_1, S_v)}.$$  

We can give an upper bound on the probability that the message will get lost in both sectors by squaring the last equation and thus obtaining

$$\left( 1 - \left( \sum_{k=0}^{n} P_c(k, 1)(P_t(k - w(d)l_s, t_r)) \right)^{d(S_1, S_v)} \right)^2.$$  \hspace{1cm} (25)

The complementary probability is then the lower bound for the reliability of the modified system.

Experimental evaluation  Within the experiments we assume the following properties:

- $r_{max} = 500$ the maximum transmission range is 500 meters,
- $s_l = 250$ the sector length is a consequence of the maximum transmission range,
- $N = 150$ the maximum number of nodes is 150,
- $v_n = 30$ the speed of stations is 30 m/s, which is about 110 km/h,
- $t_g = 1$ the time between the generation of two new messages is at least 1 second,
- $p_r = 0.9$ the reliability of the radio communication,
- number of bg-slots is 15 for every second.

Figure 19 shows the reliability as a function of the number of nodes within one sector.
8 Conclusions

We have presented a complete system for constructing warning system for a mobile ad-hoc communication environment. Within the system a set of algorithms has been shown and evaluated which are usable in other ad-hoc communication systems where low-bandwidth is a great concern. The choice of one of the presented solutions can be made basing on the specific requirements for a given implementation.

References


